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Electro-Magnetically Tuneable Filter

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Annotation

Commencement of my study at gymnasium opened for me an opportunity to elaborate an SSA essay. A lecture on terahertz (THz) spectroscopy drew my attention on the open house day at the Czech Republic Academy of Science Institute of Physics (IP). Therefore, I made a decision to develop my interest even more, and I approached Dr. Kužel, the Head of the IP terahertz laboratory. THz spectrum zone is currently in limelight of very intensive scientific research already offering applications in medicine, goods and persons checking procedures, and others. It is quite likely that telecommunications technologies will reach THz frequencies in several years; thus, it is important to work on developing optical elements that would allow communication. Dr. Kužela's team, for instance, researches into fast modulators and tuneable filters of THz radiation.

The objective of my essay was to design a simple filter based on properties of fotonic structures and tuneable by electromagnet. The filter will contain a material whose properties in THz zone can be changed, e.g. through temperature or electric field. A temperature-tuneable filter using a similar principle has already been developed. As compared to temperature-tuned filter, filter tuned by electric field has several advantages, but also disadvantages:

- Speed is its major advantage;
- Lower sensitivity to tuning and transmittance, as compared to temperature-tuneable filters, appears currently as its major disadvantage.

Therefore, I have decided, with the assistance of Dr. P. Kužel as my consultant, to research into this problem and provide its description, or to offer a solution.

Acknowledgement

I would to thank Mendels Grammar school for an opportunity to elaborate an SSA essay and Dr. Petr Kužel for his patience and will to pass on his experience.

Affirmation

I affirm that I have elaborated this essay by myself and cited all the literary sources I have used.

In Opava on March 20, 2006.
Vojtěch Šimetka

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Author’s signature

CONTENTS:

1. Electromagnetic Waves Propagation and Complex Notation

In this essay, I will deal with propagation of electromagnetic waves in various environments; I also will try to characterise light transmittance through various stratified structures and reflectivity on these layers. In describing, I will use the notion of planar waves that can be described in the form of following equations:

$$E = E_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r}) \qquad (1)$$

$$H = H_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r}) \qquad (2)$$

Electromagnetic waves have an electric and magnetic component bound to each other, so I will only work with the electric component from now on. **k** is the wave vector determining the direction of the wave propagation. Mathematically, it is convenient to use so-called complex number notation. We can write:

$$E = E_0 e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} = E_0 [\cos(\omega t - \mathbf{k} \cdot \mathbf{r}) + i \sin(\omega t - \mathbf{k} \cdot \mathbf{r})] \qquad (3)$$

This representation is very useful for most mathematical calculations and introduces intuitively the notion of so-called wave amplitude E_0 and wave phase ($\omega t - \mathbf{k} \cdot \mathbf{r}$). However, we have to keep in mind that only the real component of the expression (3) has a physical meaning (the imaginary component was used just from mathematical reasons).

The wave equation expressing the relation between the wave circular frequency ω and the wave vector magnitude *k* applies for planar waves in any environment:

$$\frac{\omega N}{c} = k \qquad (4)$$

c is the velocity of light; *N* is the refractive index of the given environment. Any electromagnetic wave polarises the environment in which it propagates; this relates to the vibratory movement of electrons and charged nuclei in reaction to the wave's electric field. Thus, a part of the light energy is put in the given environment in the form of polarisation that propagates along with the wave and can be converted back to light on the sample's exit face. Let's accept as the fact that the greater is the polarisation, the slower is then the propagation of polarisation and light. And the refractive index just describes this interaction; the speed of light in the given environment determines the refractive index magnitude, and it can be proved that it directly relates to the energy volume put in the environment.

$$N = n - i\kappa \qquad (5)$$

Then, per equations (5) and (4), it applies for the given frequency ω that the wave vector is complex

$$k = k` - ik`` = \frac{\omega n}{c} - i\frac{\omega\kappa}{c} \quad (6)$$

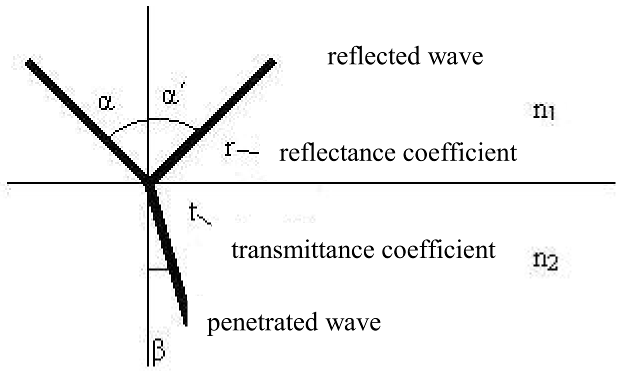
Then, it ensues from the equation (3) for propagation (e.g. in the *z* direction):

$$E = E_0 \exp[i\omega(t - n z / c)] \exp(-\omega\kappa / c) \quad (7)$$

It is immediately apparent from the equation (7) that κ characterises the extent of the light absorption in the given environment (it called the absorption index) and that the light phase propagates through this environment with speed $v = c/n$.

2. Propagation Close to Interface and in Stratified Structures

The wave falling on the interface under any angle partially reflects back on the interface, and partially penetrates it. It generally applies that the angle of incidence (α) equals to the angle of reflection (α′). Also, the angle α determines how much energy gets in the environment (transmittance) and how much of it reflects (reflectance). Then the magnitude of reflectance and transmittance further depends on the refractive index of the given environment. .



(8)

$$n_1 \sin\alpha = n_2 \sin\beta$$

$$\alpha = \alpha`$$

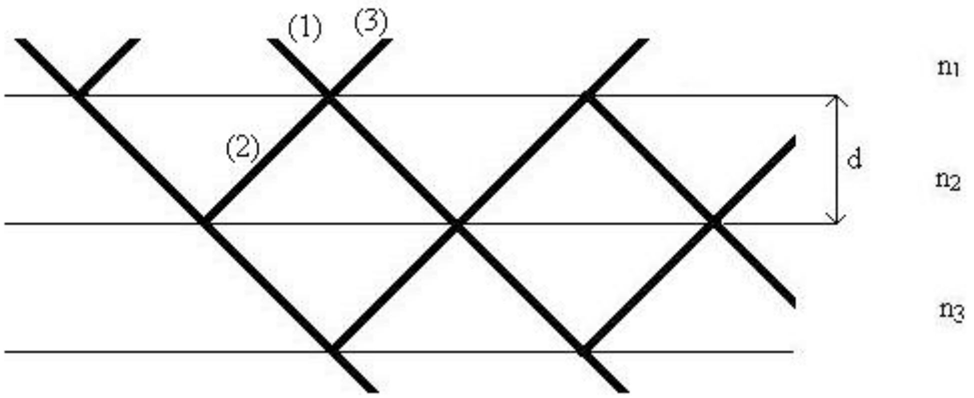
It ensues from Figure (8) that if n_2 is greater than n_1 (reflection on the optically thicker environment), the angle of incidence is greater than the angle of refraction and vice versa. Using so-called Fresnel's formulas [1], it can be calculated what part of the energy of light will be reflected and what part will go through. The simplest form of Fresnel's formulas is for perpendicular fall (I will be dealing with throughout this essay from now on):

$$r = \frac{n_1 - n_2}{n_1 + n_2}$$

$$t = \frac{2n_1}{n_1 + n_2} \qquad (9)$$

It ensues from equation (9) that for reflection of the wave on the optically thicker environment ($n_2 > n_1$) the reflectance coefficient r is negative, which means that the phase of the wave's electric field will be changed by π [because exp(iπ) = -1]. On the contrary, for reflection on the optically thinner environment ($n_2 < n_1$) is $r > 0$ and, consequently, the wave's phase remains unchanged.

Especially interesting is propagation of light in stratified structures where, in addition to the refractive index, the important role is played by interference (or superposing waves reflected on single layer interfaces).



(10)

I only deal with the perpendicular fall in this essay; non-zero angle of incidence is shown in the figure just for completeness. Figure (10) shows the complexity of superposing waves on single interfaces. E.g., the wave marked as number (1) will, after reflection on the interface 1-2, superpose with the wave marked as number (2) that originated in virtue of partial reflection on the interface between environments 2 and 3. The difference of their phases in the location of superposing will equal [using equation (7)]:

$$\Delta\Phi = 2 n_2 d\, \omega / c.\ (+\pi), \qquad (11)$$

where d is the thickness of the layer number 2. Presence or absence of the additive term +π depends on whether the phase is changed in the interface between layers 1 and 2 and 2 and 3 due to reflection on the optically thicker environment [refer to discussion after equation number (9)]. If $\Delta\Phi = 2m\pi$ (where m in an integer), constructive interference occurs and the resulting wave (3) will be reflected. If $\Delta\Phi = (2m+1)\pi$, destructive interference occurs and reflection will be suppressed.

In the optics of stratified environments, so-called quarter-waves are important where the optical thickness of the wave equals to $nd = \lambda/4$ (where $\lambda = 2\pi c/\omega$ is the wavelength of radiation). For such a wave applies that

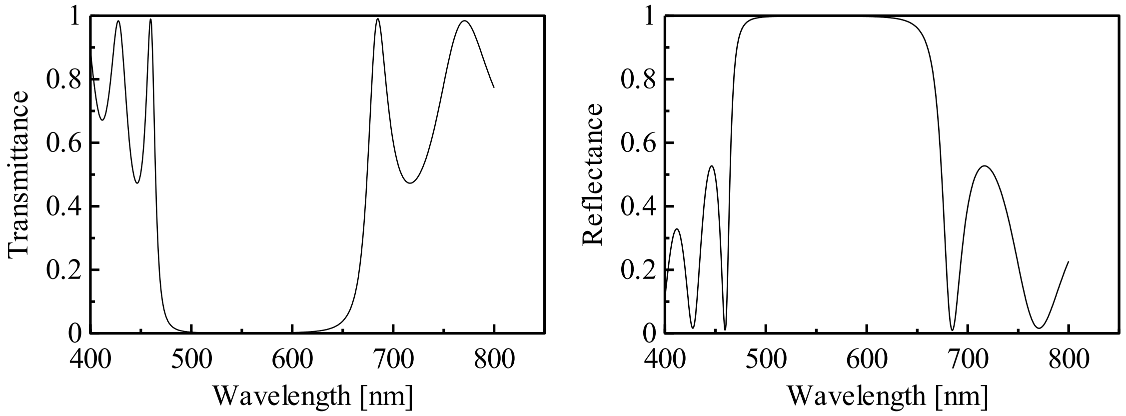
$$\Delta\Phi = \frac{\omega}{c} 2nd \ \ (+\pi) = \frac{\omega}{c} 2 \frac{2\pi c}{4\omega} \ \ (+\pi) = \pi \ \ (+\pi) \tag{12}$$

and, consequently, per the character of reflection either fully constructive, or fully destructive interference occurs. Similar situation occurs also in case of so-called half-waves when $nd=\lambda/2$. For such a wave then applies $\Delta\Phi = 2\pi\ (+\pi)$.

Generally speaking, in structures where the reflection constructive interference occurs, also the transmission destructive interference will occur, and vice versa. That way, we can design so-called antireflection or, conversely, highly reflective layers.

Transmittance and reflectans properties of stratified optical structures can be very conveniently calculated using so-called transmission matrices method described e.g. in [1]. For my work, I used the Pkgraph programme where this calculation method is directly implemented allowing calculations for any configurations of fotonic structures with any dispersion and absorption properties. As input information, I define codes for sequencing layers, thickness of single layers, their frequency-dependent refractive indexes and absorption indexes and the angle of incidence. The output information is in the form of a graph and data file stating reflectance and transmittance of the given structure.

Graphs (13) show transmittance and reflectance spectra of so-called dielectric mirror designed for the optical spectral zone. This structure consists of a series of thin layers coated on a substrate. It is a periodic arrangement, in which layers with an appropriate thickness and a low refractive index (marked as L, refractive index 1.46, thickness 94 nm) interchange with layers with a high refractive index (marked as H, refractive index 2.4, thickness 57 nm). Thanks to constructive and destructive interferences occurs so-called forbidden band as the part of the light spectrum, for which the given structure of LH…L layers is impermeable. In Figure (13), it can be found for wavelengths between 480 and 650 nm. In this zone, the structure behaves as a very good mirror.


Transmittance
0
0.2
0.4
0.6
0.8
1
400
500
600
700
800
Wavelength [nm]
Reflectance
0
0.2
0.4
0.6
0.8
1
400
500
600
700
800
Wavelength [nm]

(13)

3. THz Spectral Zone and Introduction to Fotonic Structures

Terahertz (THz) spectral zone is situated in the frequency interval of 0.1 to 10 Thz (which are frequencies 60 to 6000 times lower than the visible radiation (light) frequency). Until recently, it was very hardly experimentally available because the classical radiation sources have a very low efficiency. Fundamental works, leading to new experimental methods allowing spectroscopic measurements in Thz zone, were published in the late eighties and early nineties of the last century. Today, it is a dynamically developing scientific discipline wit a plenty of applications.

Fotonic crystals are artificially created structures with a periodic refractive index arrangement. They show so-called forbidden band, which is a frequency zone where the given structure is completely impermeable for electromagnetic radiation, although materials the structure was created of do not absorb the radiation. It is a resonance phenomenon where constructive and destructive interferences occur among waves reflected in structure's single phases. For the existence of this phenomenon is substantial that the structure's phase be comparable with wavelength of electromagnetic radiation for which it is designated. Fotonic crystals for the visible spectral zone (light) must then show periodicity in the scale of hundreds of nanometers; the period magnitude for THz spectrum is in the area of fractions of millimetres.

Inserting a defect that can be conceived as a small motif with a distinctly different refractive index may interrupt periodicity of a fotonic crystal. This way, creating so-called defective mode, which is a very narrow spectral line, can occur in the optical forbidden band where the given structure, on the contrary, becomes completely transparent and may be used e.g. as a spectral filter.

4. Material Dispersion in THz zone

The refractive index and absorption index introduced in Chapter 1 generally depend on the frequency of radiation. This phenomenon is called dispersion. The refractive index of material will generally be quite different in the optical and THz zone of electromagnetic spectrum. Similar general assertion also applies for the absorption index: a material transparent in one spectral zone may be, for instance, quite non-transparent for radiation in another zone.

Moreover, optical and Thz properties of a material may also depend on other external parameters such as temperature or applied electromagnetic field. This property of the material is called tuneability.

The objective of this work is to design tuneable fotonic structures for THz spectral zone. The following materials will be used for this purpose:

Layer L: Silica (SiO_2) with the refractive index in THz zone $n_L = 2.10$

Layer H: Cerium oxide (CeO_2) with the refractive index $n_H = 4.85$

By an interchanging arrangement of slices of these materials a fotonic structure is created with a high reflectance (similarly as for materials in the structure in Figure 13).

Inserted in this structure is so-called defective layer of strontium titanate ($SrTiO_3$, STO for short) whose properties in THz zone may easily be changed through temperature and electric field. STO shows strong so-called vibratory mode (or a vibratory condition of a specific group of atoms in the crystal grid motif). This mode has own frequency of vibrations ω_0 in THz spectral zone.

Thz radiation with this vibratory condition has a resonant nature. It means that a mode with high amplitude is induced for THz radiation with frequencies around ω_0. In other words, a large portion of electromagnetic energy in this case converts in sample's polarisation (whose large portion will converted to heat and, thus, absorbed). It ensues from reasoning in Chapter 1 that this mode will distinctively influence the refractive index in Thz zone. We can write [1]:

$$N = \sqrt{n_0^2 + \frac{f}{\omega_0^2 - \omega^2 + i\omega\gamma}} \qquad (14)$$

where ω_0 is the frequency of vibratory mode cycles, γ is its dumping (related to absorption), f is so-called oscillator's force (determines the interaction force between the radiation and the vibratory mode) and n_0 is the refractory index at high frequencies (above THz zone). We learnt from the experiment [2] that the frequency ω_0 could be influenced by temperature and electric field. Also, we know from the experiment [3] the specific dependence of the refractive index on temperature and electric field.

5. Temperature Tuning of Fotonic Structures

I used the Pkgraph program mentioned above for my work. Graph (15) shows the curve of changes in STO dispersion and absorption properties depending on temperature. I calculated this dependence using formula (14) where I cast parameters taken from source [3]:

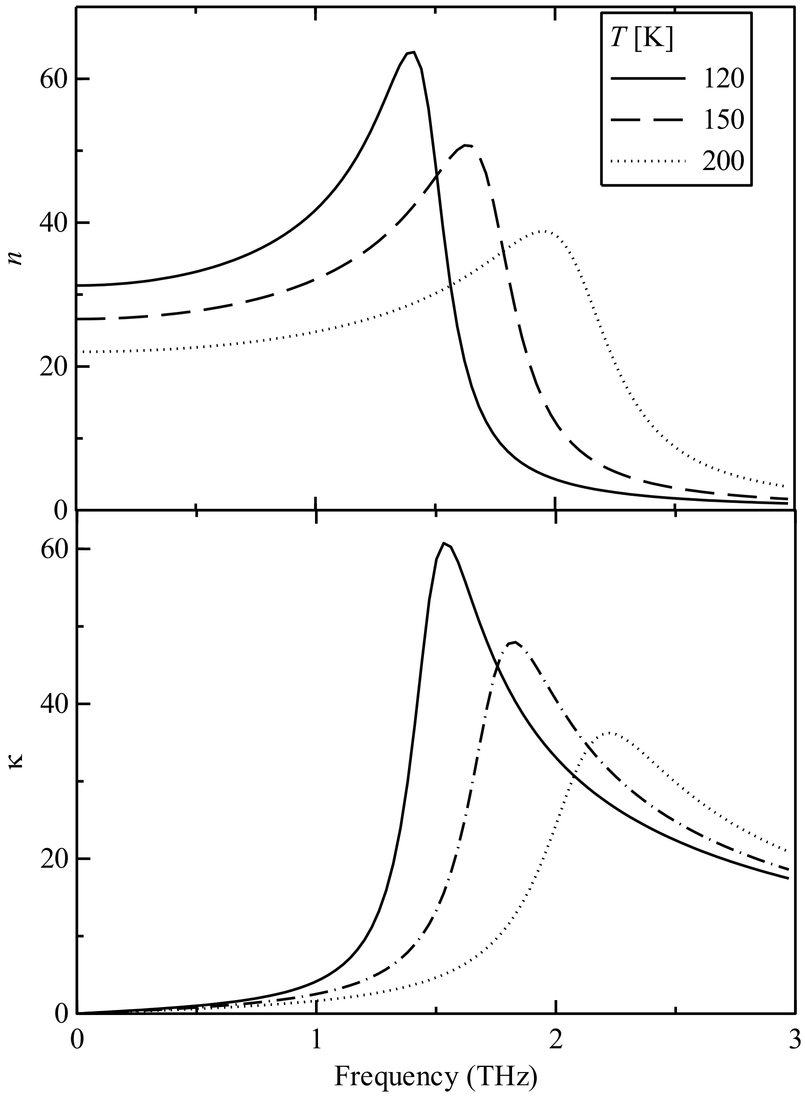
$$\omega_0[\mathrm{cm}^{-1}] = \sqrt{31.2\left(T[\mathrm{K}] - 42{,}5\right)}$$

$\gamma\,[\mathrm{K}] = -3.3 + 0{,}094\ \mathrm{T}$

$f\ [\mathrm{cm}^{-2}] = 2.34 \times 10^{6}$

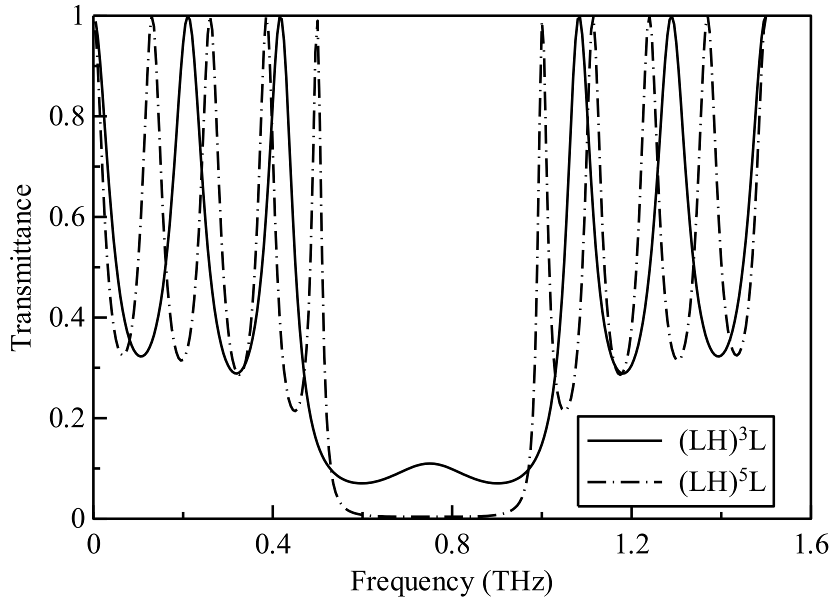
$n_0 = 3.1$

Regarding fotonic structures, we are mostly interested in the zone of 0.1 and 0.8 Thz where significant changes in the refractory index occur with changing temperature, whereas the absorption index κ remains low. I, therefore, assume that right in this zone STO will be well tuneable with a low absorption. I will be designing fotonic structures just for this spectral zone.



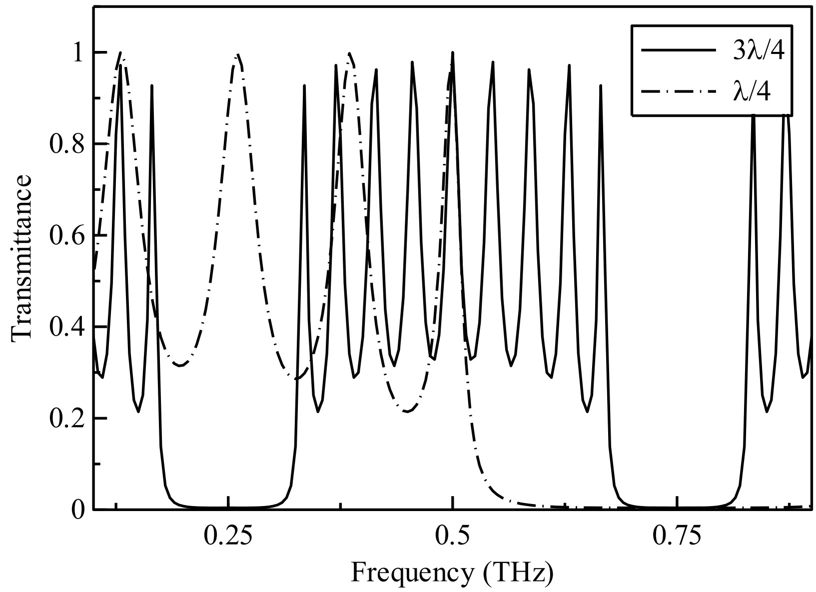
(15)

In the first stage, we designed the periodic structure without the defective layer for THz so that it has the forbidden band of frequencies around 0.75 THz (i.e. for the wavelength 0.4 mm) for us to be able to use above-mentioned materials - SiO_2 (Layer L) a CeO_2 (Layer H)). Thickness of layers corresponds to the optical thickness λ/4 is: d_H = 20.6 μm, d_L = 47.6 μm. Graf (16) shows the study of the shape and width of the forbidden band (approx. 0.6 to 1.0 THz) depending on the number of double-layers; specifically for $(LH)^3L$ and $(LH)^5L$ structures.



(16)

In Graph (17), I then explored changes of the forbidden band shape based on assumption of using thicker layers; Figure specifically shows data for optical thickness of both layers equal to 3λ/4 (i.e. d_H=61.8 μm, d_L= 143 μm). It is apparent that the forbidden band is narrower around 0.75 THz; per contra, another forbidden band occurs around 0.2 to 0.3 Thz. I will use both these forbidden bands for tuning after insertion of the defective layer into the structure.

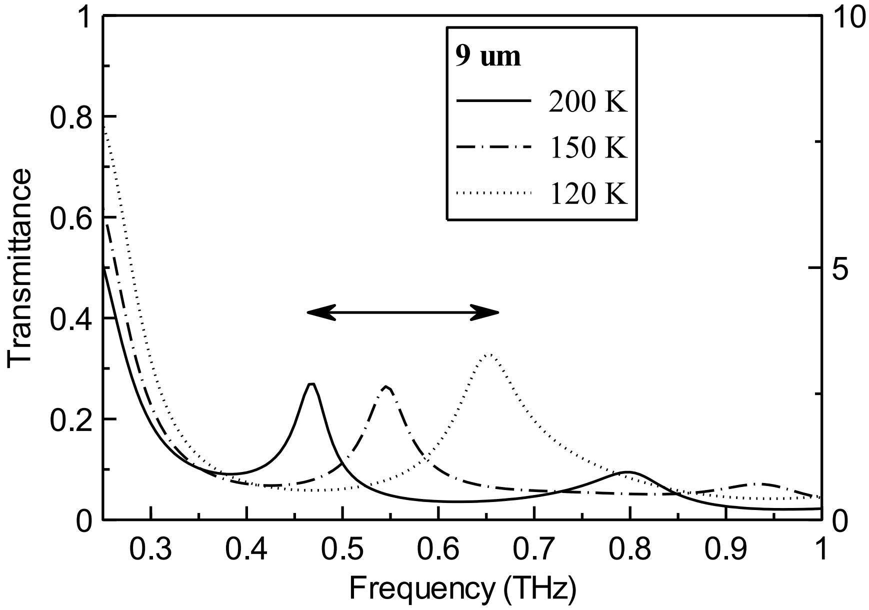


(17)

Graphs (18, 19, 20) show the transmittance spectrum of the very filter depending on temperature. It is a LHLHL**S**LHLHL (where S is the defect, STO). It is apparent that a spectral line with higher transmittance, whose frequency moves with changing temperature as shown by the arrow in each figure, appears in the forbidden band.

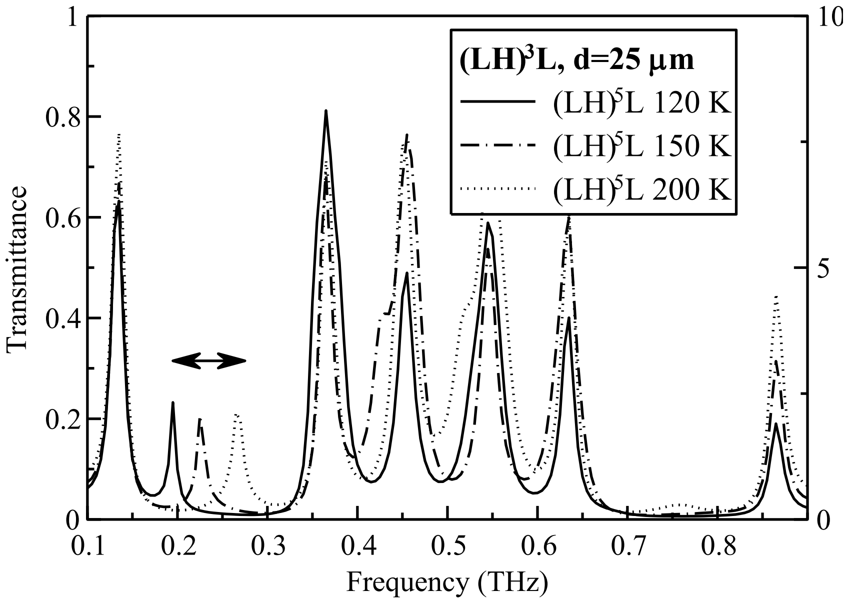
Parameters of structures in each figure are as follows:

(Figure 18)	(Figure 19)	(Figure 20)
d_L = 47.6 μm	d_L = 142.7 μm	d_L = 47.6 μm
d_H = 20.6 μm	d_H = 61.8 μm	d_H = 20.6 μm
d_S = 9 μm	d_S = 25 μm	d_S = 45 μm



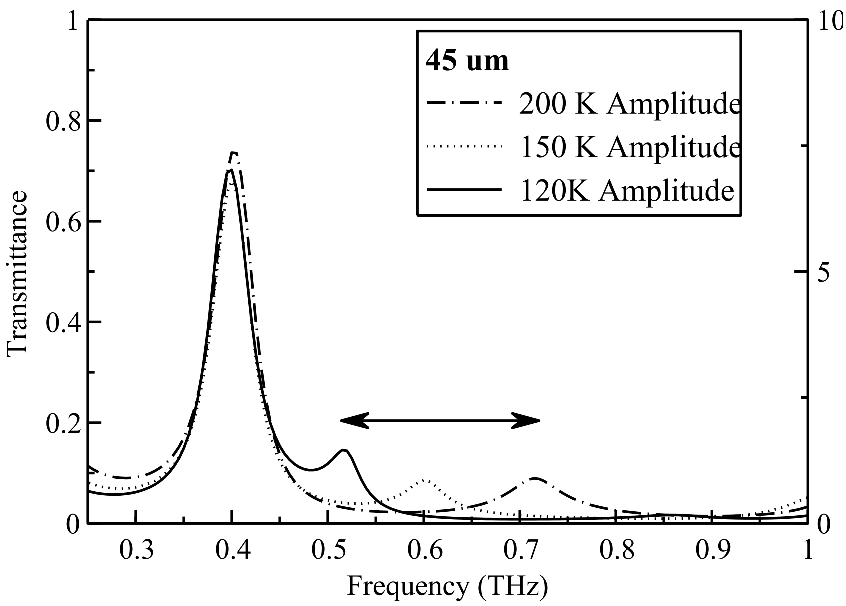
(18)

The STO slice 9 μm thick is very difficult to produce; therefore, the graph is just an attempt of how the spectrum of such a structure could be like. The lower is d_S, the higher is namely the tuneability, in this case in the range of 0.45 to 0.65 THz.



(19)

In temperature tuning, it is useful to work in the range of 0.15 to 0.35 Thz (the first forbidden band). The STO transmittance in this spectral zone is still relatively high. It is apparent that the width of spectral lines in the second forbidden band for the defective thickness of 25 μm is very low in comparison to the first forbidden band: this illustrates the increasing absorption in accordance with Figure (15).

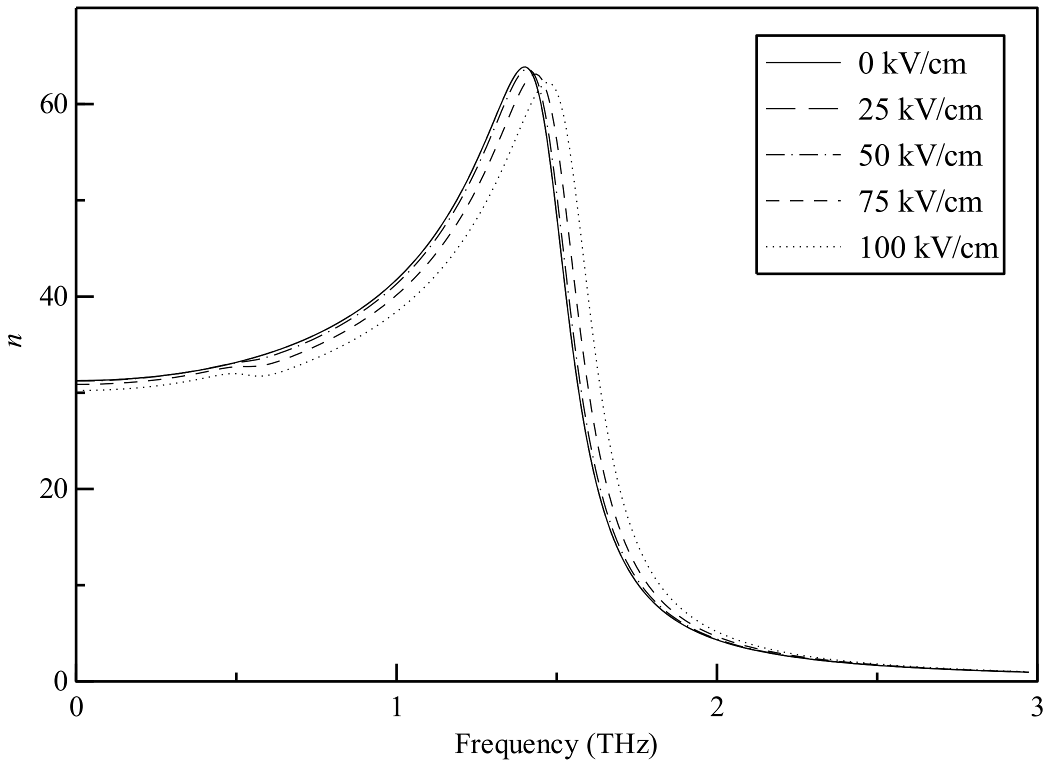


(20)

The zone of 0,5 to 0.75 THz is not by far so useful as the zone around 0.25 Thz. It is mostly caused by the absorption magnitude, although the frequency displacement of the peak in the spectral zone is higher.

6. Tuning Fotonic Structures by Electromagnetic Field

Tuning by electromagnetic field gives much worse results for the tuneability than those for temperature tuning. It is caused by the fact that a new mode appears in THz spectrum around 0.5 Thz [3] with a higher electromagnetic field applied to the sample — this mode is called the "second mode" in graphs. Microscopic origin of this mode is not explained so far, nevertheless, experiments show that its contribution lowers the tuneability of the refractive index in the electric field directly in the zone below 0.5 Thz [4]. It means that tuning around 0.25 Thz – thus in the zone of the first forbidden band in our structure, refer to Figure (17) - with values of the electromagnetic field up to 30 kV/cm (where the existence of this mode is proved experimentally and its characteristics quantitatively determined) is practically disallowed. Figure (21) shows spectra of the refractive index depending on the electric field applied for the temperature of 120 K. Figure (22) then shows the substantial part of this dependence in the enlarged scale. It is apparent from the figure that certain tuning could occur for very high values of the electromagnetic field applied, nevertheless, dependencies calculated for these values are merely extrapolation of measurement results for lower fields and they are not fully reliable for the present.



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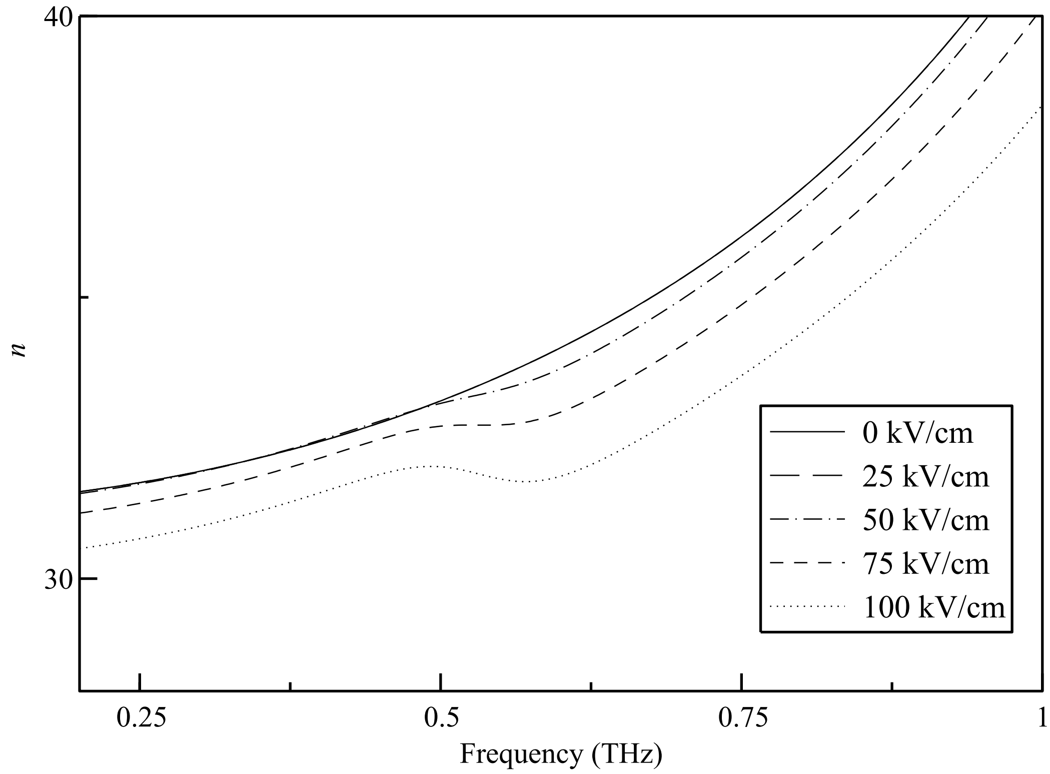
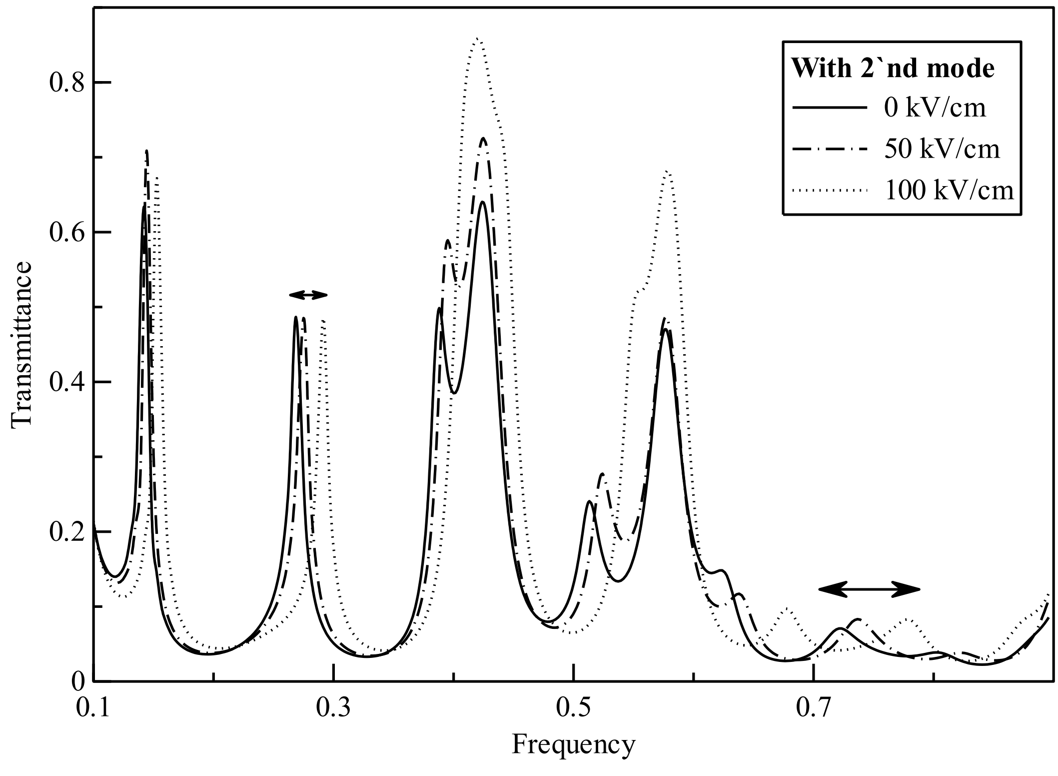


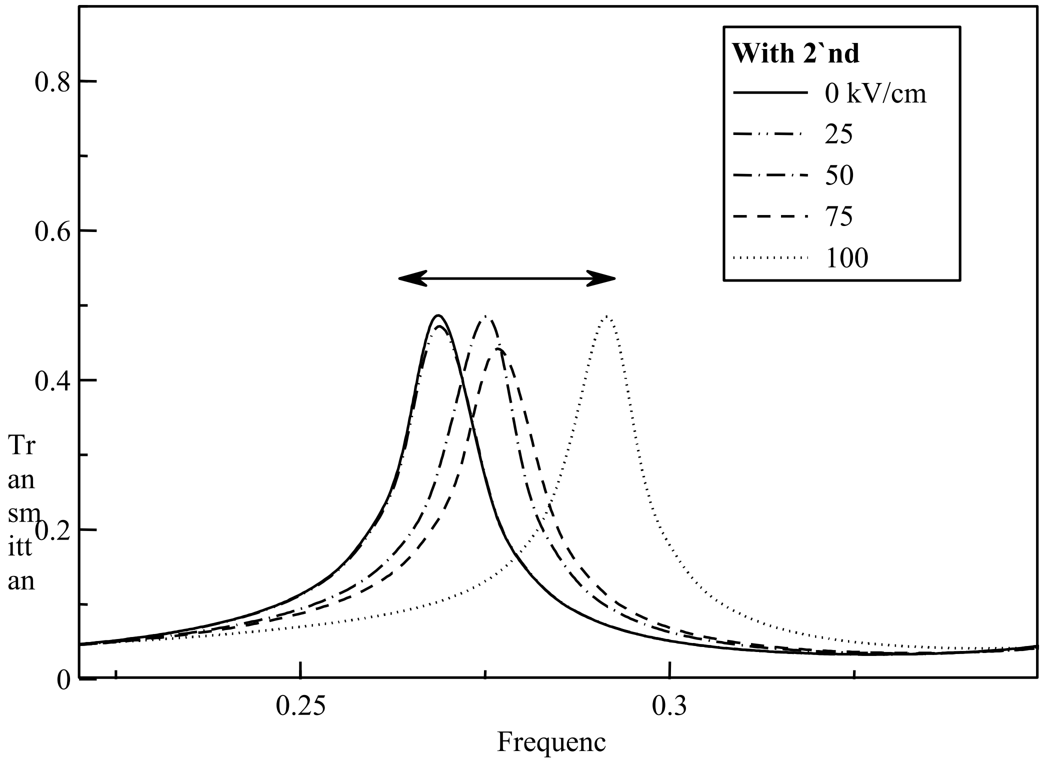
(22)



(23)

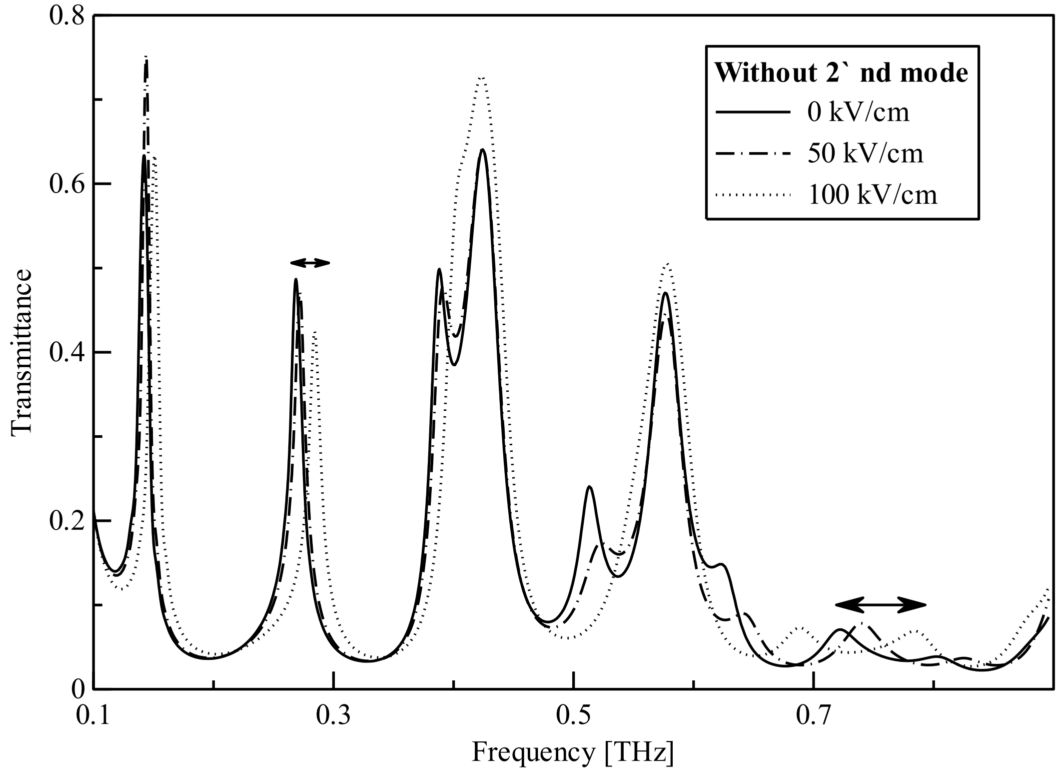
Because of this additional contribution that is not appearing in temperature tuning, we are forced to follows the tuneability in the second forbidden band from Figure (17) — i.e. in the vicinity of the frequency 0.75 Thz but where a relatively high absorption already occurs. It manifests itself by a low intensity and relatively high half-width of the tuned peak which is situated between 0.7 and 0.8 THz. This phenomenon can be observed in Figure (23) I have calculated for a structure with the following parameters: $(LH)^5L$ d_S = 27 μm, T = 120 K.

Figure (24) shows spectra of this structure around 0.25 Thz in an enlarged scale. The observed peak does not move in the range of electric field values up to 25 kV/cm (which is experimentally verified). Tuning the frequency occurs for higher values of electric field, however, I remind that this is just an extrapolation of experimental results.



(24)

For comparison, I also created graphs of transmittance spectra of structures studied that we could obtain when the additional mode would not make itself felt — Figure (25, 26). It is apparent that in this case tuning could be possible around 0.25 Thz even for low values of electric field.



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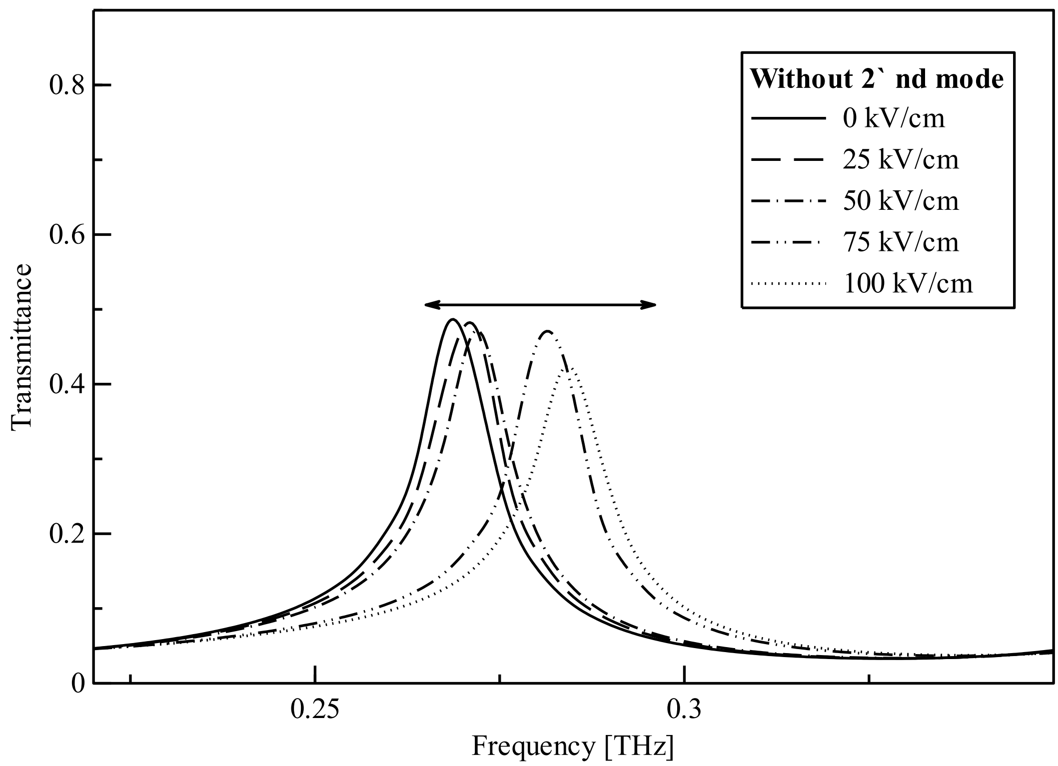

(26)

Conclusion

Filter tunable by elektromagnetic field has a good tunability only for higher voltage than 30 kV/cm, but the data of tunability for higher voltage is not experimentally available. In the course of problem solving I try to allow for a claim utility of a filter. That I choose the material with the best atributes for tunability in THz spectrum.

This work was for me a big gains, because I impruve in my physical experience. I learned using of PKgraph program, found a new possibilities of physics and learned new things on computer work. I learned about physicial and chemical properties of $SrTiO_3$, SiO_2 and CeO_2.

Literary sources used

[1] M. Born and E. Wolf, *Principles of Optics*, 7th ed. (University Press, Cambridge, 2003).
[2] P. Kužel, F. Kadlec, H. Němec, R. Ott, E. Hollmann, and N. Klein, *Appl. Phys. Lett.* 88, 102901 (2006).
[3] P. Kužel and F. Kadlec, *Comptes Rendus de l'Académie des Sciences – Physique*, (2007), v tisku.